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The application of improved cascade EM algorithm to multi-user detection for CDMA system

Li Zhihui[2],Zhu Yuhua[2,3],Zhen Tong[2]

Abstract. The linear convergence is a serious disadvantage of the EM algorithm and its extension which affects the application of them. In order to improve the convergence speed, the improved Cascade EM algorithm was proposed in this paper. In the Aitken-delta Cascade EM (Aitken-delta CEM) algorithm, the realization of this unique improved algorithm consists of the Aitken-delta algorithm and Cascade PersonNameProductIDEM algorithm. TheEM algorithm. The improved algorithm is applied in Multi-User Detection (MUD), the results show that the improved Cascade EM algorithm has good performance. Compared with the proposed algorithm, the convergence speed is faster than traditional algorithm. So the Aitken-delta algorithm combine with the Cascade EM algorithm will achieve a stable convergence, while convergence speed can be accelerated.

Key words. Multi-user detection (MUD),aitken-delta algorithm,cascade EM,aitken-delta cascade EM.

1. Introduction

In the code division multiple access (CDMA) system, multi-user detection is served as one of the key technologies. Multi-user detection (MUD) is the one standard of 3G, which can effectively reduce the multiple access interference (MAI) and increase the system capacity, and Multi-user detection has long been a study of the popular at home and abroad, but analysis the problem of high complexity and robustness that limits their practical application. With the development of Multi-user detection and some algorithms that are not particularly high performance complex have been proposed, multi-user detection will be applied in the chmetcn-

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[2]Workshop 1 - College of Information Science and Engineering, Henan University of Technology, Zhengzhou, 450001, Henan, P.R. China. e-mail: zhihui_li511@sina.com

[3]Workshop 2 - Yellow River Conservancy Techinical Institute, Kaifeng, 475004, Henan, P.R.

vTCSC0NumberType1NegativeFalseHasSpaceFalseSourceValue4UnitNameg4G system. It is proposed originally by K. S. Schneider in 1979. This method can eliminate the multi-access interference and alleviate the near-far effect effectively. Verdu put forward the best multi-user detection method in 1986[1]. Its performance is best, but its complexity grows exponentially with the increase in the number of users. So the method is difficulty to be achieved in practical engineering application.

Many researchers put forward different kinds of sub-optimal multi-user detection. The complexity is reduced. But the performance is significant lower than the optimal multi-user detection algorithm. In 1977, Dempster, Laird and Rubin (DLR) firstly proposed the Expectation-Maximization (EM) iterative algorithm model in their research paper (Ref.[2]); in Ref.[3], H. Poor apply the EM algorithm to Multi-User Detection (MUD) in 1996, and the other paper such as references [4] and [5] about EM application in MUD. The Expectation-Maximization (EM) algorithm and its extension are commonly used for the missing data in recent years, because of their simple operation, and approaching maximum likelihood estimation of optimal performance, they are supported by the researchers of all ages.

They are so popular because they could simply constantly tend to global optimal solution by iterative step. In exceptional circumstances, even if it is easy to find the full-likelihood expectation, it is difficulty to achieve the maximization of the full-likelihood expectation. Fortunately, the Cascade EM algorithm is appropriated for solving the difficult problem. It works with an intermediate complete data specification, and performs intermediate EM steps at some iteration. There is, however, the disadvantage for EM algorithm and its extensions that the convergence speed is linear. So the linear convergence is a serious defect of Cascade EM algorithm itself which affects its application. In this paper, background noise chose to Gaussian noise, the Cascade EM algorithm and Aitken-delta method are used to Multi-User Detection (MUD) in order to accelerate the convergence speed.

This paper is organized as follows. In section 2, we introduced notation and provide sufficient conditions for the Cascade EM algorithm and Aitken-delta algorithm. The algorithm theories also were introduced in this chapter.

In Section 3, firstly, the model of the multi-user detection system was introduced in this chapter. The system was based on the framework of CDMA and the Gaussian noise. And then, we present the Accelerated improved EM algorithm applied in MUD. In Section 4, we described the simulation results and conclusions, and show that the improved algorithm accelerates the convergence of the traditional algorithms.

2. The Theories of the Cascade PersonNameProductIDEM and ImprovedPersonNameProductIDEM and Improved AlgorithmEM and Improved Algorithm

2.1. Cascade EM Algorithm

Let$Z$denote the observed data vector, possessing the probability density$f_Z(z;\theta)$indexed by the vector parameters$\theta \in \Theta$, where$\Theta$is a subset of the Euclidean p-space [6].

Given an observedZ = z, the Maximum Likelihood (ML) estimate is the value of θ that maximizes the log-likelihood function, that is

$$\max_{\theta\in\Theta} f_Z(z;\theta) \Rightarrow \hat{\theta}_{ML}. \tag{1}$$

Suppose that Zcan be regarded as being incomplete, and there are two hierarchies of complete data specifications $Y$and Xsuch that

$$H_1(Y) = Z, H_2(X) = Y \tag{2}$$

Where $H_1(\cdot)$and$H_2(\cdot)$ are noninvertible (many-to-one) transformations. Express densities

$$\begin{array}{l} f_Y(y;\theta) = f_Z(z;\theta) f_{Y/Z=z}(y;\theta), \\ \forall H_1(y) = z \end{array} . \tag{3}$$

Where$f_Y(y;\theta)$ is the probability density of$Y$, and$f_{Y/Z=z}(y;\theta)$is the conditional probability density of Y given$Z = z$.Taking the logarithm,

$$\log f_Z(\mathrm{z};\theta) = \log f_Z(z;\theta) - \log f_{Y/Z=z}(y;\theta), \forall H_1(y) = z. \tag{4}$$

Taking the conditional expectation given Z = z at a parameter value $\theta^{'}$ (that is multiplying both sides of (4) by $f_{Y/Z=z}(y;\theta^{'})$ and integrating over$y(z) = \{y|H_1(y) = z\}$,

$$\log f_Z(z;\theta) = Q_Y(\theta,\theta^{'};z) - P_Y(\theta,\theta^{'};z) \tag{5}$$

In complete analogy with (4), we also have

$$\begin{array}{l} \log f_Y(y;\theta) = \log f_X(x;\theta) - \log f_{X/Y=y}(x;\theta), \\ \forall H_2(X) = Y \end{array} \tag{6}$$

Taking the conditional expectation givenY $= y$at a parameter valueθ^*

$$\log f_Y(y;\theta) = Q_X(\theta,\theta^*;\mathrm{y}) - P_X(\theta,\theta^*;y) \tag{7}$$

Taking the conditional expectation of (7) givenZ $= z$ at a parameter value$\theta^{'}$

$$Q_Y(\theta,\theta^{'};z) = Q(\theta,\theta^{*},\theta^{'}) - P(\theta,\theta^*,\theta^{'}). \tag{8}$$

Implies:$Q(\theta,\theta^{*},\theta^{'}) > Q(\theta^{*},\theta^{*},\theta^{'})$,so

$$Q(\theta,\theta^{'},z) > Q(\theta^{*},\theta^{'},z). \tag{9}$$

The relations in (9) form the basis to the Cascade EM (CEM) algorithm. Denote by $\hat{\theta}^{i}$the current estimate ofθ. Then, the next parameter estimate is defined by:

E-step: Compute

$$Q_Y(\theta,\hat{\theta}^i;z) = E_{\hat{\theta}^i}\left\{\log f_Y(Y;\theta)/Z = z\right\}. \tag{10}$$

M-step: Solve

$$\underset{\theta'}{Max}\, Q_Y(\theta, \hat{\theta}^i; z) \Rightarrow \hat{\theta}^{i+1} \tag{11}$$

Via: Set$\hat{\theta}_0^{i+1} \equiv \hat{\theta}^i$.For $m = 0, 1, \cdots, M-1$ do:
Intermediate E-step: Compute

$$\begin{array}{l} Q(\theta, \hat{\theta}_m^{i+1}, \hat{\theta}^i) = E_{\hat{\theta}^i} \bullet \\ \left\{ E_{\hat{\theta}_m^{i+1}}[\log f_X(X;\theta)/Y]/Z = z \right\} \end{array}. \tag{12}$$

Intermediate M-step: Solve$\underset{\theta}{Max}\, Q(\theta, \hat{\theta}_m^{i+1}, \hat{\theta}^i) \Rightarrow \hat{\theta}_{m+1}^{i+1}$. Set

$$\hat{\theta}^{i+1} \equiv \hat{\theta}_M^{i+1}. \tag{13}$$

In practice, we may want to limit the number Mof internal cycles, depending on the index $i$of full cycles. Since$\hat{\theta}_{m+1}^{i+1}$is obtained by maximizing $Q(\theta, \hat{\theta}^{i+1}, \hat{\theta}^i)$then by (14) it immediately follows that

$$\log f_Z(\mathrm{z};\ \hat{\theta}_{m+1}^{i+1}) > \log f_Z(\mathrm{z};\ \hat{\theta}^i). \tag{14}$$

For $m = 0, 1, \cdots, M-1$.Therefore, the sequence $\left\{ \hat{\theta}^i \right\}$ $i = 0, 1 \cdots$monotonically increases the l\likelihood function.

2.2. Aitken-delta Cascade EM Algorithm

The Aitken-delta method is first proposed by Masahiro Kuroda, Michio Sakakihara, and Zhi Geng , which is a non-linear method for accelerating the convergence and it is particularly powerful for linear sequence convergence [10].

First, we describe the Aitken-delta method. Let$\{\phi^r\}_{r\geq 0}$be a scalar sequence which converges toϕ^*. For the scalar sequence$\{\phi^r\}_{r\geq 0}$, the Aitken-delta method generates a sequence$\{\phi^r\}_{r\geq 0}$by

$$\dot{\phi}^r = \phi^r - \frac{(\phi^{r+1} - \phi^r)^2}{(\phi^{r+2} - 2\phi^{r+1} + \phi^r)}. \tag{15}$$

For convergence of the scalar sequence$\{\phi^{\mathrm{r}}\}_{\mathrm{r}\geq 0}$, Traub [8] has provided the lemma.

Lemma. If $\{\phi^{\mathrm{r}}\}_{\mathrm{r}\geq 0}$ converges to a stationary point ϕ^* as$r \to \infty$, then $\{\ \dot{\phi}^{\mathrm{r}}\ \}_{\mathrm{r}\geq 0}$ generated by formula (10) converges to the same station pointϕ^*.

To compare the speed of convergence of the sequence $\{\ \dot{\phi}^r\ \}_{\mathrm{r}\geq 0}$from the Aitken-delta method with that of the equence$\{\phi^{\mathrm{r}}\}_{\mathrm{r}\geq 0}$, we use the following notion of Brezinski and Zaglia [9].

Definition. Let $\{\hat{\phi}^r\}_{\mathrm{r}\geq 0}$be a scalar sequence obtained by applying an extrapolation method to$\{\phi^{\mathrm{r}}\}_{\mathrm{r}\geq 0}$.Assume that $\lim\limits_{t\to\infty} \phi^r = \lim\limits_{t\to\infty} \hat{\phi}^r = \phi^*$.If $\lim\limits_{t\to\infty} \frac{|\hat{\phi}^r - \phi^*|}{|\phi^{r+2} - \phi^*|} = 0$, then we say that sequence$\{\phi^{\mathrm{r}}\}_{\mathrm{r}\geq 0}$ converges toϕ^* faster than $\{\phi^{\mathrm{r}}\}_{\mathrm{r}\geq 0}$ or that the extrapolation method accelerates the convergence of$\{\phi^{\rm r}\}_{\rm r\geq 0}$.Traub proved that the Aitken-delta method acceleration the convergence of $\{\phi^{\rm r}\}_{\rm r\geq 0}$ in the sense that

$$\lim_{t\to\infty}\frac{|\dot{\phi}^{r}-\phi^{*}|}{|\phi^{r+2}-\phi^{*}|}=0. \tag{16}$$

In order to accelerate the convergence of the sequence$\{\phi^{r}\}_{r\geq 0}$ obtained by the ECM algorithm, we apply Aitken-delta acceleration to generate the sequence$\{\dot{\phi}^{r}\}_{r\geq 0}$. The Aitken-delta acceleration for the ECM algorithm is presented as follows.

Let $\theta^{0}=\dot{\theta}^{0}$denote the initial value.

E-step: Using$\theta^{r}$and the observed frequencies, calculate the expected marginal counts for each generator.

M-step: find θ^{r+1} by using Esq.(13).

Aitken-delta acceleration: Calculate$\varphi^{r-1}=\varphi(\theta^{r-1})$,$\varphi^{r}=\varphi(\theta^{r})$,$\varphi^{r+1}=\varphi(\theta^{r+1})$, where$(\theta^{r-1},\theta^{r},\theta^{r+1})$is obtained at the previous M-steps. Generate a vector $\dot{\varphi}^{r-1}=(\dot{\varphi}_{i}^{r-1})_{i=1,\ldots,d-1}$ from

$$\dot{\varphi}_{i}^{r-1}=\varphi_{i}^{r-1}-\frac{(\varphi_{i}^{r}-\varphi_{i}^{r-1})^{2}}{\varphi_{i}^{r+1}-2\varphi_{i}^{r}+\varphi_{i}^{r-1}},i=1,2,\ldots,d-1. \tag{17}$$

Aitken-delta accelerated algorithm improved the M-step, and its core thought is that the parameter estimates again after Cascade EM iteration estimation, the concrete step will be detailed introduced in next Multi-user detection application.

3. The Cascade EM Algorithm and Improved Algorithm Applied in MUD

3.1. Multi-user Detection

The k-th receive signal is:

$$r(t)=\sum_{k=1}^{K}A_{k}(t)g_{k}(t)b_{k}(t)+n(t). \tag{18}$$

In the formula,δ means the amplitude of the k-th signal;$i++$ means spread spectrum waveform of the k-th signal, and the value is± 1; $b_k$means the k-th user data, the value isb_{opt}; $n(t)$is background noise that is selected by different type of the noise.

Suppose$y_k$is the output of the k-th matched filter and its expression is:

$$y_{k}=\int_{0}^{T}r\left(t\right)g_{k}\left(t\right)dt, 1\leq k\leq K. \tag{19}$$

This result can be obtained by expansion:

$$y_k = \int_0^T \left(\sum_{k=1}^{K} A_k(t) g_k(t) b_k(t) + n(t) \right) g_k(t)\, dt = A_k b_k + MAI_k + n_k \tag{20}$$

It can be known clearly that the first item is the data that the k-th user wants to receive; the second item is multiple access interference (MAI) which is generated by other users; and the third is noise[9].

In order to facilitate processing and analysis, formula (20) can be expressed as matrix form:

$$y = RAb + n \tag{21}$$

$y = [y_1, y_2, \ldots, y_K]^T$is the set of the output signals of matched filters. $b = [b_1, b_2 \ldots, b_K]^T$ is the user symbol vector, $R$is a symmetric correlation matrix with$K \times K$dimension ($\rho_{i,k} = \rho_{k,i}$). $A = diag(A_1, A_2 \ldots, A_K)$, A means the amplitude of the received signal which is a diagonal matrix. $z = [z_1, z_2, \ldots, z_k]^T$, it is a complex-valued vector with independent real and imaginary components and covariance matrix equal to$\sigma^2 R$.The purpose of multi-user detection is to detect users' signals θ^i from the output signals of the matched filter$\theta^{t+1} = \theta^t - l''(\theta^t|x)^{-1} l'(\theta^t|x)$..

3.2. Cascade EM Detection

In Synchronous DS-CDMA system, the cascade EM multi-user detection is where adding the expectation maximum function in the classic detector. Select here decorrelation detection, and get the initial estimate$\{\phi^{\mathrm{r}}\}_{\mathrm{r}\geq 0}$, then apply cascade EM iterative algorithm, suppose the k-th user information bit estimate isϕ^*,$\{\phi^{\mathrm{r}}\}_{\mathrm{r}\geq 0}$means the i-th iterative value of cascade EM[13].

As in ref. [3] the expectation complete likelihood function is;

$$Q(b_k|b_k^i) = \frac{A_k^2}{2\sigma^2}(-(b_k)^2 + 2b_k \frac{1}{A_k}(y_k - \sum_{j \neq k} R_{kj} A_j b_j)) \tag{22}$$

In above formula$b_j = E\{b_j|r(t), b_k = b_k^i\}$.Using Bayes' formula, the E-step update is defined by

$$b_j = \tanh(\frac{A_j}{\sigma^2}(y_j - R_{jk} A_k b_k^i)). \tag{23}$$

Maximize expectation:

$$b_k^{i+1} = \arg\max Q(b_k|b_k^i). \tag{24}$$

Because of $\hat{b} = \{\pm 1\}$we obtain the simple formula$b_k^{i+1} = sgn(y_k - \sum_{j \neq k} R_{kj} A_j b_j)$ as the M-step.

Via: Set

$$\mathrm{b}_{k_0}^{i+1} \equiv b_k^i \tag{25}$$

We define normal deviation ratio is$Norm = \frac{||b_{km}^{i+1} - b_{km}^{i}||}{||b_{km}^{i}||}$ while we analysis iterative convergence . Repeat E-step and M-step iterations, and don't stop the loop until Norm is small enough.

3.3. 3.3. Aitken-delta Cascade EM (Aitken-delta CEM) Detector

As in ref. [3] the expectation complete likelihood function is;

$$Q(b_k|b_k^i) = \frac{A_k^2}{2\sigma^2}(-(b_k)^2 + 2b_k\frac{1}{A_k}(y_k - \sum_{j\neq k} R_{kj}A_j b_j)) \tag{26}$$

In the formula, $\dot{b}^i$is the i-th iteration of the Aitken-delta parameter estimates, and $b^{i+1}$is the (i+1)-th estimate from b^{i}via Cascade EM (CEM) algorithm[15]. The Aitken-delta Cascade EM (Aitken-delta CEM) algorithm concrete steps

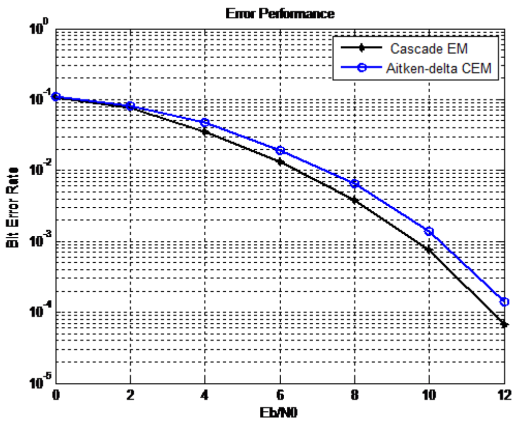

Fig. 1. The BER curve of three algorithms in Gaussian noise

4. Simulation results and Conclusions

In the DS-CDMA system, applying Aitken-delta Cascade EM(Aitken-delta CEM) accelerated algorithm, and we select 5 users, 1000 information bits, 31-bit gold spread-spectrum code, and the user power partial value is 12. Compare the Aitken-delta Cascade EM (Aitken-delta CEM) algorithm with the Cascade EM and standard EM algorithm simulation results show that: the improved algorithm has good BER performance curve that trend to the cascade EM algorithm (Figure 1); contrast Figure 2 shows the improved algorithm has faster convergence than the Cascade EM algorithm (Figure 2). So, the Aitken-delta algorithm combine with the Cascade EM algorithm will achieve a stable convergence, while convergence speed can be improved.

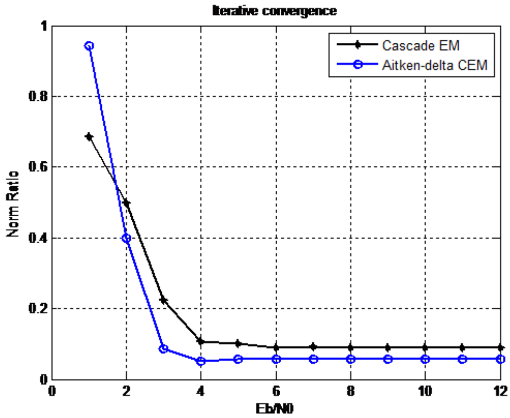

Fig. 2. Iterative convergence of three algorithms in Gaussian noise

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